THE ELUDATION OF A CRITIC ON THE CORE-PERIPHERY MODEL - THE ANALYTIC SOLUTION –

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One of the major critiques of Paul Krugman's standard core-periphery model (1991), which model forms the bottom of so called 'The New Economic Geography', is the impossibility of his analytic solution, reason for which difficult digital simulations are enforced. Getting a function and an analytic solution enables a better description of the process of agglomeration and at the same time, the presentation of the authors, with title of novelty, of graphics on the logarithmic scale, which aligns as a complementary form to the traditional ones. The adaptation and the presentation of the model with a software help, as Maple computer program, permits also the complete understanding of the solvable analytic model.

Keywords: *Paul Krugman's model, Spatial equilibrium, Analytic settled model.*

Introduction

It has not passed a relatively long time since the article entitled 'Increasing Returns and Economic Geography' has been published, but along with the indisputable merit of author's contribution, it is enforced the sensitization of researchers throughout the world about a new area from economic science- The New Economic Geography one.

The area's literature has expanded vertiginously due to contributions of numerous researchers, among which we remember: Fujita, Krugman and Venables (The Spatial Economy -1999), Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud (Economic Geography and Public Policy -2003), Fujita and Thisse (Economie des Viles et de la Localisation -2003), which have tried to answer to some pragmatic aspects from real contemporary economy, as: the globalization, the integration, the unification of Germany, the urban agglomeration, the commercial policies fundamentation.

But with all these merits, Paul Krugman's model has not been spared of critiques, which had reproached the difficulty of its solution, feasibly only through digital simulations and inferentially not getting a function, which would have contribute to the understanding of centrifugal and centripetal forces that functionate in this model and, also to the registration of some 'catastrophes' from economic point of view. The present model is mainly based on Michael Pfluger's contribution (see the work 'A Simple, Analytically Solvable, Chamberlian Agglomeration Model'), which has in sight the works of Forslid and Ottaviano (2002) and Flam and Helpman (1987).

The Model

The model is consisted of two countries: the domestic and the foreign one (noted with _ext), two production factors: the work (L) and the human capital (K), two sectors: the Industry (M) and the agriculture (A).The work is presumably mobile between sectors, while the countries are presumably holding preferences, technology and identical commercial costs. On long term the human capital is presumably international mobile while the work is not.

The good produced in agriculture is homogenous, made under a perfect competition, with constant scale incomes, utilizing only the work factor and also is presumably commercialized without costs. This good is selected as the denominator and also is produced in both countries.

The industrial sector (M), which produces the industrial good under a monopolistic competition utilizes both the production factors, respectively the work (L) and the human capital (K) to produce differentiated industrial goods with a linear cost's function , the work being the variable in time component, while the human capital –the fixed one, the only human capital unit being necessary. The good industrial commercialization develops commercial costs of 'iceberg' type under a samuelsonian sense.

The Consumers Component

In economy exist $L+K$ holders of production factors, each of them offering an unit from the work factor and the human one, in return of ***s*** and ***r***.

The preferences of each consumer are:

$$U = \alpha + \ln C_M + C_A$$

$$C_M = \left(\sum_{i=0}^{N} m_i^{\frac{\sigma-1}{\sigma}} + \sum_{j=N}^{N\_ext} m_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

$$\alpha > 0; \sigma > 1$$

where *CM* is the aggregate consumption from industrial good, *CA* is the consumption from agricultural good, *mi (mj)* is the quantity spent from the illustrative internal variety, respectively from external variety *j* , *N* and *N_ext* is the number of varieties produced by the domestic and foreign country, *σσ* is the substitution elasticity from industrial varieties.

It will be registered a budgetary compulsion in the shape of:

$$Y = PC_M + C_A$$

$$P = \left(NP_i^{1-\sigma} + N\_ext(\tau P_j)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tau > 1$$

where *Y* is the income, *P* is a perfect index of price, *Pi (Pj)* is the price fixed by a domestic (foreign) company. Transport costs (or commercial ones in a larger sense) are constant and equal with *τ*, implying that only *1/τ* from a unit of a variety of foreign industrial good to reach to destination, in such a way that the price supported by a consumer for an imported variety to be *τPj*. The maximization of utility determines the demand's functions and the undirect utility *UI*:

$$C_M = \frac{\alpha}{P}; \; C_A = Y - \alpha \; ; \; m_i = \alpha P_i^{-\sigma} P^{\sigma-1};$$

$$m_j = \alpha (\tau P_j)^{-\sigma} P^{\sigma-1}$$

$$UI = -\alpha \ln(P) + Y + \alpha(\ln(\alpha) - 1)$$

The Producers Component

With a convenient choose of units and nominating with *LA* the work factor, production function of the agricultural good will be *XA=LA*. Perfect competition causes that the price of this good to be equal with the marginally cost and respectively with the medium (average) one, and the choice of agricultural good as denominator, the salary rate ***s*** is equal with unit, that is: ***s=1.***

The equilibrium for a variety I from industrial good is determined by the relation:

$$X_i = (L + K)m_i + (L\_ext + K\_ext)\tau \; m_i\_ext$$

where X_i is the production, and m_i *(m_i_ext)* is the demand of a representative domestic (foreign) consumer. Each variety is presumably produced by a single company. With *s=1* and $L_i=cX_i$, ***c*** being a positive constant, the marginally cost is equal with ***c*** constant.

The fixed cost which registers the solicitation of a unit from human capital is equal with ***r***.

If we are defining with P_i*(P_ext)* the producer's price which is applied to a domestic(foreign) consumer, and with Π_i the profits of a representative company from domestic country, we will obtain:

$$\Pi_i := (P_i - c)(L + K)x_i + (P\_ext_i - c) \cdot$$
$$\cdot (L\_ext + K\_ext)\tau x\_ext_i - r$$

the prices that maximize the profit are equal with a constant relation, as $\frac{\sigma}{\sigma-1}$ over the marginally cost, that is:

$$P_i = P\_ext = \frac{c\sigma}{\sigma - 1}$$

The income obtained by the human capital *r* is adjusting to assure that in equilibrium the profit should be *0* (zero). Utilizing the condition of equilibrium we obtain a relation between the production of a company X_i and the fixed cost *r*, that is:

$$X_i = \frac{r(\sigma - 1)}{c}$$

The Short Term Equilibrium

On short term the human capital is motionless between the countries, so that:

N=k ; N_ext=K_ext

In free trading conditions, utilizing relations 2, 3, 5 and 6 and the converse for foreign country, the conditions of 0 (zero) profit in domestic country and foreign one will be:

$$\sigma r = \frac{\alpha (L + K)}{K + \Phi K\_ext} + \frac{\Phi \alpha (L\_ext + K\_ext)}{\Phi K + K\_ext}$$

$$\sigma\ r\_ext = \frac{\Phi\,\alpha\left(L + K\right)}{K + \Phi\,K\_ext} + \frac{\alpha\left(L\_ext + K\_ext\right)}{\Phi\,K + K\_ext}$$

$$0 \le \Phi = \tau^{(1-\sigma)} \le 1$$

The income of human capital r and r_ext from domestic and foreign country in equilibrium on short term is obtained solving relation 8, after that is obtained the production of a company from relation 7. The whole industrial sector will employ $\boldsymbol{N\ c\ X_i = N\ r\ (\sigma - 1)}$ working units which are smaller than L, in such a way that both sectors to be active. This condition can be described by the inequality:

$$\alpha < \frac{\rho\,\sigma}{\left(2\rho + 1\right)\left(\sigma - 1\right)}$$

$$\rho = \frac{L}{K + K\_ext}$$

The Long Term Equilibrium

On long term the human capital is international mobile and the owners of this factor will shift to the country where the indirect utility is higher. It can be defined an indirect utility as:

$$UI - UI\_ext = \alpha\ \ln\left(\frac{P\_ext}{P}\right) + \left(r - r\_ext\right)$$

which substituted will determine that:

$$UI - UI\_ext = \frac{\alpha}{1-\sigma}\ln\left(\frac{\lambda\Phi + \left(1-\lambda\right)}{\lambda + \left(1-\lambda\right)\Phi}\right) + \frac{\alpha\left(1-\Phi\right)}{\sigma}\left(\frac{\rho + \lambda}{\lambda + \left(1-\lambda\right)\Phi} - \frac{\rho\_ext + \left(1-\lambda\right)}{\lambda\Phi + \left(1-\lambda\right)}\right)$$

$$\rho = \frac{L}{K + K\_ext};\ \rho\_ext = \frac{L\_ext}{K + K\_ext}$$

$$UI - UI\_ext = \frac{\alpha\ \ln\left(\dfrac{\dfrac{K\tau^{1-\sigma}}{K + K\_ext} + 1 - \dfrac{K}{K + K\_ext}}{\dfrac{K}{K + K\_ext} + \left(1 - \dfrac{K}{K + Kext}\right)\tau^{1-\sigma}}\right)}{1-\sigma} +$$

$$+\frac{1}{\sigma}\left(\alpha\left(1-\tau^{1-\sigma}\right)\left(\frac{\dfrac{L}{K + K\_ext} + \dfrac{K}{K + K\_ext}}{\dfrac{K}{K + K\_ext} + \left(1 + \dfrac{K}{K + K\_ext}\right)\tau^{1-\sigma}} - \frac{\dfrac{L\_ext}{K + K\_ext} + 1 - \dfrac{K}{K + K\_ext}}{\dfrac{K\tau^{1-\sigma}}{K + K\_ext} + 1 - \dfrac{K}{K + K\_ext}}\right)\right)$$

It is interesting to study the function of indirect utility difference between the domestic and the foreign country depending on the human capital weight of the domestic country for different levels of transport costs, that is *DIF =UI-UI_ext* depending on the dimension of $\lambda = \dfrac{K}{K + K\_ext}$, for τ's different dimensions. The long term equilibrium will be registered for λ's dimensions for which *DIF =UI-UI_ext=0*, as in case where $\lambda = 1/2$, putting it differently, in the situation where the two countries are equal in which regards the endowment with human capital. But an other important aspect, which is determined by the size of transport costs is the equilibrium's steadiness. This model, as all models from the New Economic Geography area, contains two agglomeration forces. There is one agglomeration force which is developing through the supply relation, respectively a geographical entity (town, region, country) with a higher human capital weight which will have a larger industrial sector and a lower index of price. There is also one agglomeration force which is developing through the demand relation because an extension of human capital weight in a region will imply a larger market which will grow the profitability of companies and which will be reflected in an attraction of the human capital. If those forces of agglomeration act as a centrifugal force, there will also be a centripetal force which will oppose the agglomeration and which has as a result the production's dispersal, which is acting through transport costs.

In order to understand the way how these forces act and dictate the long term equilibrium, it is useful to study the function: *DIF=UI-UI_ext* for different levels of transport costs. We will consider that: **σ=6 ; α=0.3 ; p=1; p_ext=1**. Thus we will have a function with two variables, λ and τ which are graphically presented in **Figure 1**:

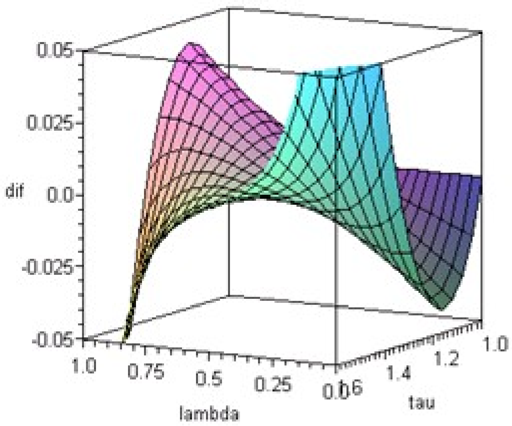

Figure 1-The difference of international utility depending on human capital's weight from a lambda country and the transport tau costs.

For $\tau \in$ *[1.1..1.5]* we will have the following graphic:

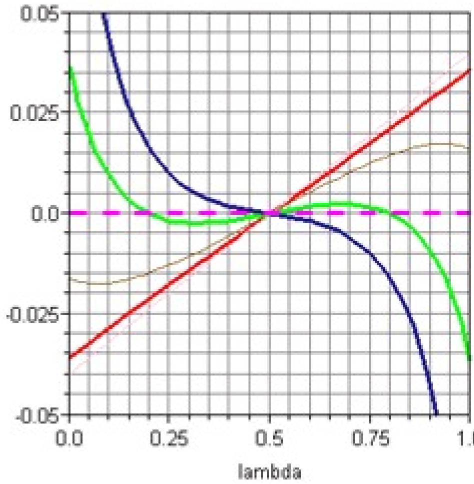

Figure 2- The difference of international utility for transport tau costs between 1.1 and 1.5

From Figure 2 we choose three transport costs levels, which determine different actions of international utility difference, respectively:

a. tau= 1.1 reduced transport costs ;
b. tau= 1.4 intermediary transport costs ;
c. tau= 1.5 high transport costs

which are presented in Figure 3, 4 and 5 and in Figure 6 they are reunited:

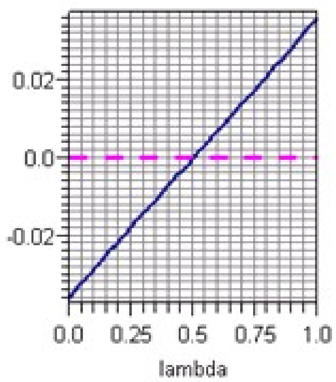

Figure 3- The difference of international utility, low transport costs tau=1.1 normal scale

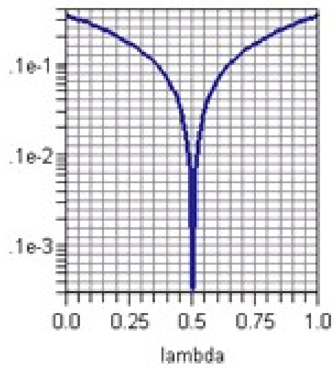

Figure 3- The difference of international utility, low transport costs tau=1, logarithmic scale

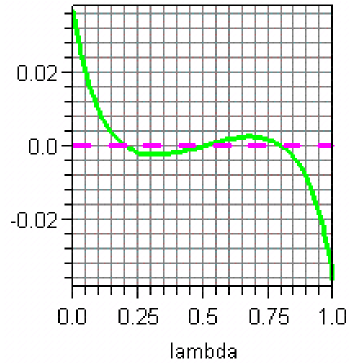

Figure 4-The difference of international utility, intermediary transport costs tau=1.4 normal scale

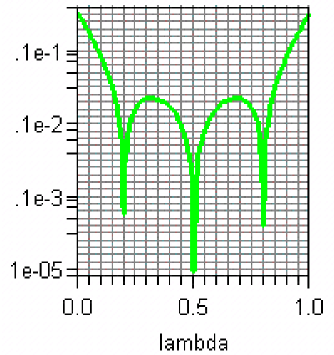

Figure 4-The difference of international utility, intermediary transport costs tau=1.4 logaritmic scale

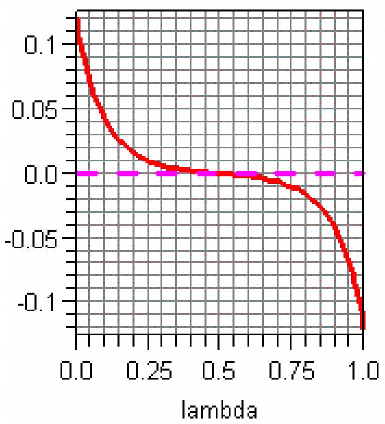

Figure 5-The difference of international utility, high transport costs tau=1.5 normal scale

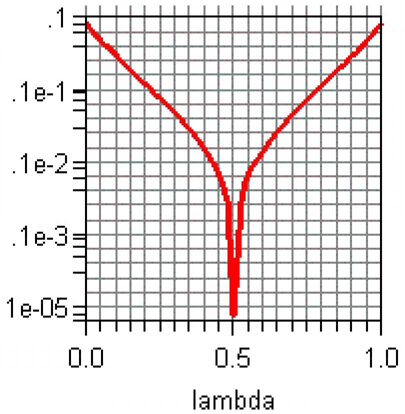

Figure 5-The difference of international utility, high transport costs tau=1.5 logaritmic scale

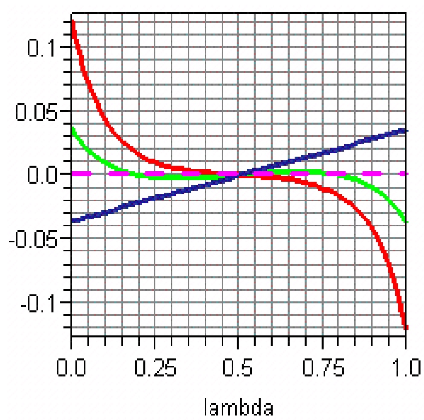

Figure 6- The difference of international utility, different transport costs tau=1.1 blue; tau=1.4 green; tau=1.5 red normal scale

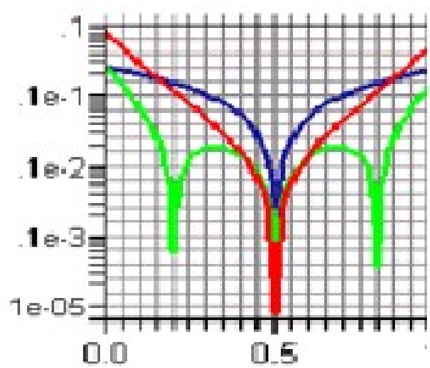

Figure 6- The difference of international utility, different transport costs tau=1.1 blue; tau=1.4 green; tau=1.5 red logaritmic scale

The analysis of graphics points out the situation that for high transport costs, $\tau=1.5$, the difference of international utility is eventless decreasing and the equilibrium is balanced, for $\lambda=0.5$ it is a stable one. It is registering this feature as through a displacement from a dispersal equilibrium of a single human capital holder from foreign country to the domestic one, increasing *λ's* proportion it would register a decrease in the difference of utility and it would return in the foreign country re-establishing the dispersal equilibrium.

In intermediary transport costs case $\tau=1.4$, the balanced equilibrium continues to be a stable one which is accompanied by two equilibriums with instable partial agglomerations.

In a low transport costs case, the balanced equilibrium becomes an instable one, as the circulation of a single human capital holder from the foreign country to the domestic one, putting it differently the dimension of λ, will contribute to an increasing of the difference in international utility and the migratory person will not have anymore a motivation to return to equilibrium contributing to an agglomeration of the human capital in a particular country.

For low transport costs $\tau=1.1$, the difference of international utility function registers an increasing grade implying that the balanced equilibriums cause a full agglomeration in a human capital country , in this case $\lambda=1$ domestic country.

It can be determined the level of transport cost, for which the balanced equilibrium is modifying from a stable to an instable one, which suppose that $\partial/\partial\lambda(UI-UI\_ext)=0$ for $\lambda=1/2$, where we consider $\sigma=6$ and $p=1$ and we will obtain an equation where the convenient solution is: $\boldsymbol{\tau c = 1.454061151}$.

Putting it differently, for transport costs lower than 1.45 the balanced equilibrium is instable and for bigger proportions, the balanced equilibrium is a stable type one.
An other critical dimension of transport cost is the one which registers a full agglomeration of human capital in one of the two countries, which is supposing $\partial/\partial\lambda(UI-UI\_ext)=0$ *for* $\lambda=$, *for which the solution is:* $\boldsymbol{\tau f= 1.279441369}$.

This means that for transport costs lower than 1.27 it is registering a full agglomeration of human capital in one of the two countries.

The two critical values of the transport costs can be seen with the help of a diagram which is frequently used in the New Economic Geography:

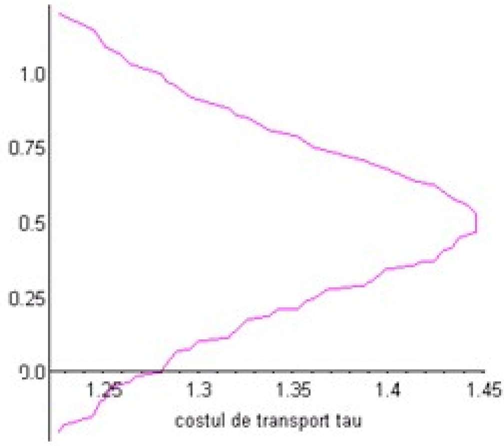

Figure 7- The bifurcation diagram

If it is not registering transport costs, $\tau=1$ the difference of international utility does not count because it is equal to 0 and the human capital holders are indifferent about the country where they move.

Conclusions

The presented model acts as Paul Krugman's standard one and it is analytic solvable and allowing to study the transport cost changes impact in determining the agglomeration or dispersal process.
We consider that the empirical analysis on different countries, which start to be achieved in the New Economic Geography, will be in concordance with model's statements, proving its utility.

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